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Feedback Control of Space Robot Attitude by Cyclic Arm Motion

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A method of controlling the attitude of a space robot using its arm motion is considered. The proposed method can make a large attitude change by the cyclic motion of the arm. First the arm path is planned based on the small attitude change when the arm moves along a closed path. Then the path is modified by feeding back the attitude error during the arm motion. Numerical simulations have been executed to verify the feedback law using a space robot model with an arm with six degrees of freedom.

Nomenclature

$E(\boldsymbol{w})$	= Euler parameters whose vector part is a very small vector $\boldsymbol{w}$
$\boldsymbol{I}_m$	= $m \times m$ unit matrix
N	= total number of arm rotations
n	= arm degrees of freedom
$S(\epsilon)$	= scalar part of Euler parameters ϵ
$\boldsymbol{U}_k$	= coordinates transformation matrix from body coordinates to inertial coordinates at the end of the kth rotation of arm
$V(\epsilon)$	= vector part of Euler parameters ϵ
ϵ	= Euler parameters that express attitude of satellite-fixed coordinates (body coordinates) with respect to inertial coordinates
ϵ_k	= values of ϵ at the end of the kth rotation of arm
$\hat{\epsilon}_k$	= desired values of ϵ_k
$\tilde{\epsilon}_k$	= Euler parameters that express differences between $\hat{\epsilon}_k$ and ϵ_k; $\epsilon_k^+ \hat{\epsilon}_k$
ϵ_0	= initial values of ϵ
θ_i	= relative rotation angle of joint i
$\boldsymbol{\omega}$	= angular velocity of body coordinates with respect to inertial coordinates

Introduction

ROBOTS working in space have been actively studied in recent years.[1–7] A free-flying space robot differs from a robot fixed on the ground because the satellite body is moved by the motion of the manipulator arm. We discuss the possibility of the attitude control of a space robot using its arm motion.

Several methods have been proposed for this kind of control.[3–8] Changing the satellite attitude gradually by repeating the arm rotations has attracted a lot of interest because it can generate a large attitude change.[5–8] In this paper, the attitude control of a satellite by repeated arm rotations is also considered. Previous methods for this kind of control are related to the path planning of the arm under the assumption that the space robot model is exactly known. However, these methods have not mentioned how to converge the attitude error that occurs during the arm motion along the path when model errors or disturbances exist.

We propose here a method of modifying the arm path by feeding back the attitude error between the actual attitude and its desired value at the end of each rotation of the arm. The proposed method first plans the arm path based on predictions of a small attitude change when the arm moves along a closed path in body coordinates. Then it modifies the path by feeding back the attitude error between the actual attitude and its desired value at the end of each rotation of the arm. This method has the advantage of obtaining the desired attitude change with simple calculations even when the space robot model is not exactly known. Numerical simulations have been executed to verify the feedback law using a space robot model with an arm with six degrees of freedom.

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Path Planning of Arm

Attitude Change of Satellite by One Rotation of Arm

For a space robot with an arm composed of rotational joints, we number each joint from 1 to n from the shoulder to the hand. It is assumed that no external force is exerted on the space robot during the arm motion and that the total angular momentum of the space robot is conserved at zero. We consider a case where a large attitude change of the space robot is obtained by plural rotations of the arm. First a change of the Euler parameters by one small arm rotation is derived.[4] The space robot attitude ϵ is related to its angular velocity $\boldsymbol{\omega}$ as

$$\dot{\epsilon} = \tfrac{1}{2}\Lambda(\boldsymbol{\omega})\epsilon \tag{1}$$

where $\Lambda(\boldsymbol{\omega})$ is shown in Eq. (A7) in the Appendix.[9] Note that vectors are expressed in inertial coordinates unless otherwise specified; a dot over vectors or Euler parameters indicates the time derivative of each component. As the total angular momentum of the space robot is conserved at zero, the angular velocity of the satellite body $\boldsymbol{\omega}$ is expressed by using the joint rotation rate $\dot{\theta}_i$ $(i = 1, \ldots, n)$ as

$$\boldsymbol{\omega} = -\sum_{i=1}^{n} \boldsymbol{x}_i \dot{\theta}_i \tag{2}$$

where $\boldsymbol{x}_i$ $(i = 1, \ldots, n)$ is a 3×1 vector composed of the mass and the moments of inertia of the arm and the satellite body.[4]

Next the satellite's attitude change by a very small rotation of the arm is calculated from these relations. When each joint of the arm moves from an initial posture to the same posture along a closed path, we can express the attitude change Δ_ϵ (change of ϵ) as

$$\Delta_\epsilon = -\frac{1}{2}\sum_{i=1}^{n} \int \Lambda(\boldsymbol{x}_i)\epsilon \, \mathrm{d}\theta_i \tag{3}$$

where the integration is linear integral along the joint path. Hereafter, the differential forms are introduced to make the mathematical treatment easy. Let us define the region bounded by the closed path as C and its boundary as ∂C. When the arm motion is infinitely small, the attitude change is expressed by the theorem of exterior derivative[10] as

$$\Delta_\epsilon = \int_{\partial C} \Omega = \int_C \mathrm{d}\Omega \tag{4}$$

where Ω is the following differential 1-form:

$$\Omega = \frac{1}{2}\Lambda(\boldsymbol{\sigma})\boldsymbol{\epsilon}, \qquad \boldsymbol{\sigma} = -\sum_{i=1}^{n} \boldsymbol{x}_i \, \mathrm{d}\theta_i \tag{5}$$

and d denotes the exterior derivative. From the law of the exterior derivative, we can calculate $\mathrm{d}\Omega$ in Eq. (4) as follows:

$$\mathrm{d}\Omega = \tfrac{1}{2}\Lambda(\mathrm{d}\boldsymbol{\sigma})\boldsymbol{\epsilon} - \tfrac{1}{2}\Lambda(\boldsymbol{\sigma}) \wedge \mathrm{d}\boldsymbol{\epsilon} \tag{6}$$

where $\wedge$ designates the exterior product between differential forms. To calculate the right-hand side of Eq. (6), the following relations for the solutions of Eqs. (1) and (2) are used:

$$\mathrm{d}\boldsymbol{x}_i = \sum_{j=1}^{n}\left(\frac{\partial \boldsymbol{x}_i}{\partial \theta_j} + \boldsymbol{x}_i \times \boldsymbol{x}_j\right)\mathrm{d}\theta_j, \qquad \mathrm{d}\boldsymbol{\epsilon} = \Omega \tag{7}$$

The first equation of Eq. (7) is obtained from the time derivative of vector $\boldsymbol{x}_i$ in the inertial coordinates,

$$\dot{\boldsymbol{x}}_i = \sum_{j=1}^{n} \frac{\partial \boldsymbol{x}_i}{\partial \theta_j}\dot{\theta}_j + \boldsymbol{\omega} \times \boldsymbol{x}_i \tag{8}$$

The symbol $\partial \boldsymbol{x}_i/\partial\theta_j$ is the partial differential of $\boldsymbol{x}_i$ by θ_j when θ_j and $\boldsymbol{\epsilon}$ are regarded as independent.

Substituting Eq. (7) into Eq. (6) yields

$$\Delta_{\epsilon} = \sum_{i=1}^{n}\sum_{j=1}^{n}\int_C \Lambda(\boldsymbol{v}_{ij})\boldsymbol{\epsilon}\,\mathrm{d}\theta_i \wedge \mathrm{d}\theta_j \tag{9}$$

where $\boldsymbol{v}_{ij}$ is the vector

$$\boldsymbol{v}_{ij} = \frac{1}{4}\left(\frac{\partial \boldsymbol{x}_i}{\partial \theta_j} - \frac{\partial \boldsymbol{x}_j}{\partial \theta_i} + \boldsymbol{x}_i \times \boldsymbol{x}_j\right) \tag{10}$$

and the following identities are used:

$$\mathrm{d}\theta_i \wedge \mathrm{d}\theta_j = -\mathrm{d}\theta_j \wedge \mathrm{d}\theta_i$$
$$\sum_{i=1}^{n}\sum_{j=1}^{n} \boldsymbol{v}_{ij}\,\mathrm{d}\theta_i \wedge \mathrm{d}\theta_j = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}(\boldsymbol{v}_{ij} - \boldsymbol{v}_{ji})\,\mathrm{d}\theta_i \wedge \mathrm{d}\theta_j \tag{11}$$

Because the right-hand side of Eq. (9) is a 2-form, we can integrate it and evaluate Δ_ϵ when the joint angles move along a closed path within the space of two variables. If only one variable is used to express the joint angle θ_i, the 2-form in a one-dimensional space becomes 0 and an attitude change does not occur. However, this evaluation is limited to cases of a very small closed path because the right-hand side of Eq. (9) varies with the satellite attitude. We will consider the case where the joint angle θ_i is expressed by the two variables p_1 and p_2 as

$$\theta_i = a_i p_1 + b_i p_2 + c_i \tag{12}$$

Then $\mathrm{d}\theta_i$ becomes

$$\mathrm{d}\theta_i = a_i\,\mathrm{d}p_1 + b_i\,\mathrm{d}p_2 \tag{13}$$

Substituting Eq. (13) into Eq. (9), we obtain

$$\Delta_\epsilon = \Lambda(\boldsymbol{a}^T\boldsymbol{D}\boldsymbol{b})\boldsymbol{\epsilon} \tag{14}$$

where $\boldsymbol{a}$ and $\boldsymbol{b}$ are the vectors

$$\boldsymbol{a} = [a_1 \quad a_2 \quad \cdots \quad a_n]^T, \qquad \boldsymbol{b} = [b_1 \quad b_2 \quad \cdots \quad b_n]^T$$

and $\boldsymbol{D}$ is a matrix whose (i, j) component is the vector

$$\boldsymbol{D}_{ij} = 2\boldsymbol{v}_{ij}\int_C \mathrm{d}p_1 \wedge \mathrm{d}p_2 \tag{15}$$

As Eq. (4) holds true for an infinitely small C, $\boldsymbol{v}_{ij}$ is put out of the integral in Eq. (15). The right-hand side of Eq. (14) is calculated as if $\boldsymbol{D}_{ij}$ were a component of the matrix $\boldsymbol{D}$. From Eq. (14), the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, which give the desired attitude change, can be calculated. For example, an algorithm for a solution that minimizes $\boldsymbol{a}^T\boldsymbol{a} + \boldsymbol{b}^T\boldsymbol{b}$ has been given.[4]

Attitude Change by Plural Rotations of Arm

Next we will consider the attitude change by repeating small rotations of the arm. From Eq. (14), the Euler parameters $\boldsymbol{\epsilon}_{k+1}$ become

$$\boldsymbol{\epsilon}_{k+1} = \boldsymbol{\epsilon}_k + \Lambda(\boldsymbol{a}^T\boldsymbol{D}\boldsymbol{b})\boldsymbol{\epsilon}_k = \left\{\boldsymbol{I}_4 + \Lambda(\boldsymbol{a}^T\boldsymbol{D}\boldsymbol{b})\right\}\boldsymbol{\epsilon}_k \tag{16}$$

Because the change of the Euler parameters is very small ($\|\boldsymbol{a}^T\boldsymbol{D}\boldsymbol{b}\| \ll 1$), the right-hand side of Eq. (16) can be regarded as the product of the Euler parameters whose vector part is $\boldsymbol{a}^T\boldsymbol{D}\boldsymbol{b}$ and the Euler parameters $\boldsymbol{\epsilon}_k$. Thus, Eq. (16) can be expressed as

$$\boldsymbol{\epsilon}_{k+1} = E(\boldsymbol{a}^T\boldsymbol{D}\boldsymbol{b})\boldsymbol{\epsilon}_k \tag{17}$$

Using Eq. (A4) from the Appendix, Eq. (17) is transformed as follows:

$$\boldsymbol{\epsilon}_{k+1} = \boldsymbol{\epsilon}_k\boldsymbol{\epsilon}_k^+ E(\boldsymbol{a}^T\boldsymbol{D}\boldsymbol{b})\boldsymbol{\epsilon}_k = \boldsymbol{\epsilon}_k\boldsymbol{\gamma}, \qquad \boldsymbol{\gamma} \equiv \boldsymbol{\epsilon}_k^+ E(\boldsymbol{a}^T\boldsymbol{D}\boldsymbol{b})\boldsymbol{\epsilon}_k \tag{18}$$

From Eq. (18), $\boldsymbol{\gamma} = \boldsymbol{\epsilon}_k^+\boldsymbol{\epsilon}_{k+1}$, which means $\boldsymbol{\gamma}$ are the Euler parameters expressing the attitude change from $\boldsymbol{\epsilon}_k$ to $\boldsymbol{\epsilon}_{k+1}$. The vector part $V(\boldsymbol{\gamma})$ is calculated from the definition of $\boldsymbol{\gamma}$ as

$$V(\boldsymbol{\gamma}) = \boldsymbol{U}_k^T(\boldsymbol{a}^T\boldsymbol{D}\boldsymbol{b}) \tag{19}$$

This relation is easily obtained by Eqs. (A2) and (A5) of the Appendix. Because the vector $\boldsymbol{v}_{ij}$ in Eq. (15) is expressed in the inertial coordinates, the term $\boldsymbol{a}^T\boldsymbol{D}\boldsymbol{b}$ is also expressed in the inertial coordinates. Multiplying $\boldsymbol{a}^T\boldsymbol{D}\boldsymbol{b}$ by $\boldsymbol{U}_k^T$ expresses $\boldsymbol{a}^T\boldsymbol{D}\boldsymbol{b}$ in the body coordinates. If the vector $\boldsymbol{v}_{ij}$ is expressed in the body coordinates, the value of each component depends only on the joint angles, and each component of $\boldsymbol{v}_{ij}$ expressed in the body coordinates becomes the same at the beginning of each rotation. When vector $\boldsymbol{v}_{ij}$ is expressed in the body coordinates, let matrix $\boldsymbol{D}$ be denoted by $\boldsymbol{D}_b$, and Eq. (19) becomes

$$V(\boldsymbol{\gamma}) = \boldsymbol{a}^T\boldsymbol{D}_b\boldsymbol{b} \tag{20}$$

If the change of the Euler parameters is set equal at each rotation, $\boldsymbol{\gamma}$ also becomes constant. Therefore, the desired attitude after N rotations becomes

$$\boldsymbol{\epsilon}_N = \boldsymbol{\epsilon}_0\boldsymbol{\gamma}^N \tag{21}$$

Let $\boldsymbol{\gamma}$ be replaced by unit vector $\boldsymbol{e}$ and angle ψ as

$$\boldsymbol{\gamma} = \begin{bmatrix}\boldsymbol{e}\sin(\psi/2)\\ \cos(\psi/2)\end{bmatrix} \tag{22}$$

and the following relation holds:

$$\boldsymbol{\gamma}^N = \begin{bmatrix}\boldsymbol{e}\sin(N\psi/2)\\ \cos(N\psi/2)\end{bmatrix} \tag{23}$$

Thus, $\boldsymbol{\gamma}$ can be easily obtained from $\boldsymbol{\epsilon}_0^+\boldsymbol{\epsilon}_N$, and by setting $\boldsymbol{a}$ and $\boldsymbol{b}$ to satisfy Eq. (20), we can determine the arm rotation path.

Setting Variables p_1 and p_2

When the variables p_1 and p_2 are set to form a closed path on a parameter plane, vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ can be obtained from Eq. (20) according to the initial posture of the arm. Thus N rotations of the arm are realized from Eq. (12) when the variables p_1 and p_2 move along the closed path N times. Though the time history of variables p_1 and p_2 on the path does not affect the final attitude change of the space robot, the movement of p_1 and p_2 should preferably be smooth on the closed path for a smooth arm motion.

Here we will set the closed path to move from the origin in the p_1p_2 plane and set the time history of p_1 and p_2 as follows. The variables p_1 and p_2 are set to move counterclockwise on a unit circle with the center $(-1, 0)$ (as shown in Fig. 1). In this case, the integration value of Eq. (15) becomes the area of the unit circle, π. The values of p_1 and p_2 at angle χ become

$$p_1 = -1 + \cos\chi, \qquad p_2 = \sin\chi \tag{24}$$

Note that χ is used for the linear integration and p_1 and p_2 are used for the area integration of Eq. (4).

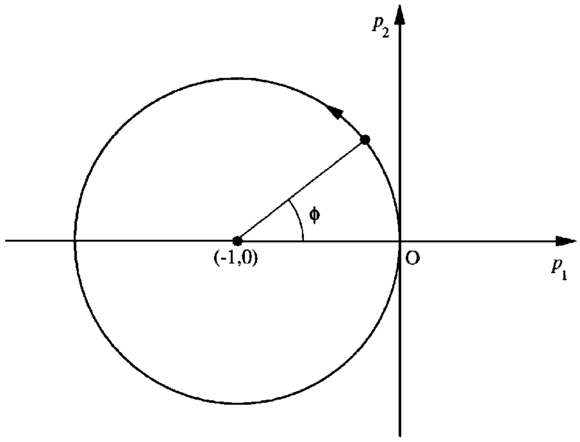

Fig. 1 Motion in p_1p_2 parameter plane.

When the arm rotates N times, we will first set $\dot{\chi}$ to increase from 0 with the constant acceleration in the first rotation, then to maintain a constant velocity from the second to the $(N-1)$th rotation, and finally to decrease to 0 with constant deceleration in the last rotation. If the time for each rotation from the second to the $(N-1)$th rotation is T, then the time for the first and the last rotations is $2T$. The angle χ at time t will be as follows:

$$\chi(t) = \frac{\pi t^2}{2T^2}, \qquad 0 \le t < 2T$$
$$\chi(t) = \frac{2\pi(t-T)}{T}, \qquad 2T \le t < NT \tag{25}$$
$$\chi(t) = \frac{2\pi(t-T)}{T} - \frac{\pi(t-NT)^2}{2T^2}, \qquad NT \le t \le (N+2)T$$

Although the movement of the variables p_1 and p_2 is not limited to the case in Eq. (25), this is used in the numerical simulations to be described.

Feedback Control of Satellite Attitude

Basic Form of Feedback Control

As already described, the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are obtained by setting the variables p_1 and p_2 to form a closed path in a two-dimensional space. Then the joint angles θ_i $(i = 1, \ldots, n)$ are determined according to Eq. (12). Therefore, the arm rotation is generated by the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. However, the desired attitude change is not exactly obtained by repeating the arm rotations N times. The reasons are as follows. First, Eq. (20) is based on an infinitesimal rotation, whereas an actual rotation is finite. The difference causes an attitude error. Second, the mass property of the space robot model cannot be identical with that of the actual space robot, which also causes an attitude error.

Here we will consider a method of converging the attitude error by feedback control during repeated arm rotations based on the solutions of Eq. (20). The proposed method is to correct the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ from the error between the attitude at the end of each rotation and its desired value.

Let us assume that the desired attitude change $\hat{\epsilon}_N$ is given and that the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are obtained to satisfy γ in Eq. (21). These vectors, $\boldsymbol{a}$ and $\boldsymbol{b}$, are replaced by their initial values $\boldsymbol{a}_0$ and $\boldsymbol{b}_0$, respectively. The following equation holds from Eq. (20):

$$\gamma = E(\boldsymbol{\eta}_0), \qquad \boldsymbol{\eta}_0 \equiv \boldsymbol{a}_0^T \boldsymbol{D}_b \boldsymbol{b}_0 \tag{26}$$

Vector $\boldsymbol{\eta}_0$ is expressed in the body coordinates because $\boldsymbol{D}_b$ is used in Eq. (26). The desired attitude change $\hat{\epsilon}_k$ at the end of the kth rotation is obtained as

$$\hat{\epsilon}_k = \epsilon_0 \gamma^k \tag{27}$$

Considering the actual attitude change from the end of the kth rotation to the end of the $(k+1)$th rotation, we express the satellite attitude ϵ_{k+1} by Eq. (18) with an additional term as

$$\epsilon_{k+1} = \epsilon_k E(\boldsymbol{\eta}_k + \boldsymbol{d}_k), \qquad \boldsymbol{\eta}_k \equiv \boldsymbol{a}_k^T \boldsymbol{D}_b \boldsymbol{b}_k \tag{28}$$

In this equation, vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ at the end of the kth rotation are replaced by $\boldsymbol{a}_k$ and $\boldsymbol{b}_k$, respectively, which do not necessarily coincide with the initial values $\boldsymbol{a}_0$ and $\boldsymbol{b}_0$. Vector $\boldsymbol{d}_k$ is a very small vector expressed in the body coordinates, which describes the difference between the actual attitude change and $\boldsymbol{\eta}_k$. As is clear from Eq. (27), $\hat{\epsilon}_{k+1}$ can be expressed as follows:

$$\hat{\epsilon}_{k+1} = \hat{\epsilon}_k E(\boldsymbol{\eta}_0) \tag{29}$$

We can define the attitude error $\tilde{\epsilon}_k$ at the end of the kth rotation as the Euler parameters from $\hat{\epsilon}_k$ to ϵ_k, that is,

$$\tilde{\epsilon}_k = \epsilon_k^+ \hat{\epsilon}_k \tag{30}$$

Using Eqs. (28) and (29), $\tilde{\epsilon}_{k+1}$ can be expressed as

$$\tilde{\epsilon}_{k+1} = \epsilon_{k+1}^+ \hat{\epsilon}_{k+1} = E^+(\boldsymbol{\eta}_k + \boldsymbol{d}_k)\tilde{\epsilon}_k E(\boldsymbol{\eta}_0) \tag{31}$$

Furthermore, the following assumptions are made.

1) Let vectors $\boldsymbol{a}_k$ and $\boldsymbol{b}_k$ be replaced by

$$\boldsymbol{a}_k = \boldsymbol{a}_0 + \delta\boldsymbol{a}_k, \qquad \boldsymbol{b}_k = \boldsymbol{b}_0 + \delta\boldsymbol{b}_k \tag{32}$$

The vectors $\delta\boldsymbol{a}_k$ and $\delta\boldsymbol{b}_k$ are then very small, and their second-order terms or higher can be neglected.

2) The vectors $\boldsymbol{\eta}_0$, $\boldsymbol{\eta}_k$, and $\boldsymbol{d}_k$ are very small, and the following holds:

$$E^+(\boldsymbol{\eta}_k + \boldsymbol{d}_k)E(\boldsymbol{\eta}_0) \approx E^+(\boldsymbol{\eta}_k + \boldsymbol{d}_k - \boldsymbol{\eta}_0) \tag{33}$$

Based on these assumptions, $\tilde{\epsilon}_{k+1}$ can be expressed as follows:

$$\tilde{\epsilon}_{k+1} = E^+(\boldsymbol{\eta}_k + \boldsymbol{d}_k)E(\boldsymbol{\eta}_0)E^+(\boldsymbol{\eta}_0)\tilde{\epsilon}_k E(\boldsymbol{\eta}_0)$$
$$\approx E^+\left(\delta\boldsymbol{a}_k^T \boldsymbol{D}_b \boldsymbol{b}_0 + \boldsymbol{a}_0^T \boldsymbol{D}_b \delta\boldsymbol{b}_k + \boldsymbol{d}_k\right)E^+(\boldsymbol{\eta}_0)\tilde{\epsilon}_k E(\boldsymbol{\eta}_0) \tag{34}$$

where the term $E^+(\boldsymbol{\eta}_0)\tilde{\epsilon}_k E(\boldsymbol{\eta}_0)$ represents the coordinates transformation of the vector part of $\tilde{\epsilon}_k$, as shown in Eq. (19). We can define the coordinates transformation matrix from $\hat{\epsilon}_k$ to $\hat{\epsilon}_{k+1}$ as $\boldsymbol{U}_\gamma$, and the following equation holds:

$$E^+(\boldsymbol{\eta}_0)\tilde{\epsilon}_k E(\boldsymbol{\eta}_0) = E[\boldsymbol{U}_\gamma V(\tilde{\epsilon}_k)] \tag{35}$$

If $\boldsymbol{u}_k$ is set as

$$\boldsymbol{u}_k = \delta\boldsymbol{a}_k^T \boldsymbol{D}_b \boldsymbol{b}_0 + \boldsymbol{a}_0^T \boldsymbol{D}_b \delta\boldsymbol{b}_k \tag{36}$$

the following equation for the vector part of $\tilde{\epsilon}_{k+1}$ holds from Eqs. (34) and (35):

$$V(\tilde{\epsilon}_{k+1}) \approx \boldsymbol{U}_\gamma V(\tilde{\epsilon}_k) - (\boldsymbol{u}_k + \boldsymbol{d}_k) \times \boldsymbol{U}_\gamma V(\tilde{\epsilon}_k) - S(\tilde{\epsilon}_k)(\boldsymbol{u}_k + \boldsymbol{d}_k) \tag{37}$$

In Eq. (37), we will assume that $V(\tilde{\epsilon}_k)$ is very small. If the second-order or higher terms of very small quantities are omitted, Eq. (37) can be simplified as follows:

$$V(\tilde{\epsilon}_{k+1}) \approx V(\tilde{\epsilon}_k) - (\boldsymbol{u}_k + \boldsymbol{d}_k) \tag{38}$$

As $\boldsymbol{d}_k$ is expressed in the body coordinates, this vector can be regarded as constant when the arm movements in each rotation are the same. Therefore, the proportional and integral control is adequate for $\boldsymbol{u}_k$ to reduce $V(\tilde{\epsilon}_k)$. On the other hand, the following equation holds from Eq. (38):

$$\boldsymbol{d}_{k-1} \approx V(\tilde{\epsilon}_{k-1}) - V(\tilde{\epsilon}_k) - \boldsymbol{u}_{k-1} \tag{39}$$

Thus, the estimated values of $\boldsymbol{d}_k$ can be replaced by $\hat{\boldsymbol{d}}_k$:

$$\hat{\boldsymbol{d}}_k = V(\tilde{\epsilon}_{k-1}) - V(\tilde{\epsilon}_k) - \boldsymbol{u}_{k-1} \tag{40}$$

Thus, $\boldsymbol{u}_k$ can be set as follows:

$$\boldsymbol{u}_k = k_P V(\tilde{\epsilon}_k) + k_I \sum_{j=1}^{k} V(\tilde{\epsilon}_j) - k_F \hat{\boldsymbol{d}}_k \tag{41}$$

where k_P, k_I, and k_F are the proportional gain, the integral gain, and the gain to compensate the error $\boldsymbol{d}_k$, respectively. The stability of the system composed of Eqs. (38), (40), and (41) can be examined by the z transformation. The characteristic equation of the system becomes

$$z\left\{z^2 + (k_P + k_I - 2)z + 1 - k_P\right\} = 0 \qquad k_I \neq 0$$
$$z + k_P - 1 = 0 \qquad k_I = 0 \tag{42}$$

The stability condition of the system is obtained by checking the condition of $|z| < 1$ as

$$k_P > 0, \qquad k_I \geq 0, \qquad 2k_P + k_I < 4 \tag{43}$$

Although the gain k_F has no relation to the stability, it is desirable to set k_F to approximately 1 because this is used for the feedforward compensation of the attitude error. Using $\boldsymbol{u}_k$ in Eq. (41), $\delta\boldsymbol{a}$ and $\delta\boldsymbol{b}$ can be given from Eq. (36) as follows:

$$\begin{bmatrix}\delta\boldsymbol{a}_k\\ \delta\boldsymbol{b}_k\end{bmatrix} = \begin{bmatrix}-\boldsymbol{b}_0^T\boldsymbol{D}_b & \boldsymbol{a}_0^T\boldsymbol{D}_b\end{bmatrix}^{\#}\boldsymbol{u}_k \tag{44}$$

where # designates a pseudoinverse matrix and the skew-symmetry of $\boldsymbol{D}_b$ is used. As the values of $\boldsymbol{a}_0$, $\boldsymbol{b}_0$, and $\boldsymbol{D}_b$ are constant, the right-hand side of Eq. (44) means the multiplication of $\boldsymbol{u}_k$ by a constant matrix, which gives this method the advantage of small calculations. When $\delta\boldsymbol{a}_k$ and $\delta\boldsymbol{b}_k$ are given from Eq. (44), $\boldsymbol{a}_k$ and $\boldsymbol{b}_k$ at the kth rotation can be set from Eq. (32). This is the basic form of the feedback control.

Making *a* and *b* Continuous

Setting the values of $\boldsymbol{a}$ and $\boldsymbol{b}$ at the $(k+1)$th rotation as $\boldsymbol{a}_k$ and $\boldsymbol{b}_k$, respectively, we can obtain the arm joint angles θ_i from Eq. (12). We will consider here the continuity of the commanded values of the arm joint angles. The arm rotations are realized by the rotations of variables p_1 and p_2 in the two-dimensional space. If the rotation of p_1 and p_2 begins at the origin, the continuity of the joint angles is maintained even though the values of $\boldsymbol{a}$ and $\boldsymbol{b}$ are different between the kth rotation and the $(k+1)$th rotation. However, the joint rates become discontinuous unless $\dot{p}_1$ and $\dot{p}_2$ are both zero at the beginning of each rotation. Therefore, it is necessary to make the joint rates zero at the beginning of each rotation. To avoid this constraint, we will give $\boldsymbol{a}$ and $\boldsymbol{b}$ as follows:

$$\begin{bmatrix}\boldsymbol{a}\\ \boldsymbol{b}\end{bmatrix} = \frac{1}{1+\upsilon s}\begin{bmatrix}\boldsymbol{a}_0 + \delta\boldsymbol{a}\\ \boldsymbol{b}_0 + \delta\boldsymbol{b}\end{bmatrix} \tag{45}$$

where s is a differentiation operator and $\delta\boldsymbol{a}$ and $\delta\boldsymbol{b}$ are set by $\delta\boldsymbol{a}_k$ and $\delta\boldsymbol{b}_k$ from Eq. (44), respectively, at the beginning of the $(k+1)$th rotation. Equation (45) means that the values of $\boldsymbol{a}_0 + \delta\boldsymbol{a}$ and $\boldsymbol{b}_0 + \delta\boldsymbol{b}$ are made smooth by a low-pass filter with a time constant υ. When the values of $\boldsymbol{a}$ and $\boldsymbol{b}$ become continuous by the filter, both the joint angles and joint rates also become continuous as the arm rotation starts at the origin of the p_1p_2 plane. It is best to set the time constant υ much shorter than one rotation time of the arm and much longer than the time constant of the joint servocontrol.

Numerical Simulations

Numerical simulations of the attitude control of a space robot by its arm motion have been executed using the algorithm just described. Figure 2 shows a space robot model. An arm with six degrees of freedom is attached to the satellite at [0.0, 1.0, 0.5] m from its center of mass. The main parameters are shown in Table 1.

The initial joint angles θ_i ($i = 1\sim6$) of the arm are [0, 60, −60, 0, −30, 0] deg, where all of the joint angles are 0 deg at the arm posture shown in Fig. 2. At the initial condition, the inertial coordinates are assumed to coincide with the body coordinates, which is expressed as xyz in Fig. 2.

The vectors $\boldsymbol{a}_0$ and $\boldsymbol{b}_0$ are given by the method in Ref. 4. We will focus here on the case where the satellite attitude is changed by 40 deg around the y axis of the inertial coordinates by 20 arm rotations ($N = 20$). Therefore, vectors $\boldsymbol{a}_0$ and $\boldsymbol{b}_0$ fitting the rotation of 2 deg around the y axis must be obtained from Eq. (23).

Table 1 Space robot parameters (nominal)

	Satellite	Manipulator arm L1	L2	L3	L4	L5	L6
Length, m	χ 2.4 × 1	0.2	1	0.5	0.5	0.1	0.2
Mass, kg	500	10	30	20	20	10	20
Moments	100	0.1	0.2	0.2	0.2	0.1	0.1
of inertia,	100	0.1	3	0.5	0.5	0.1	0.1
kgm^2	100	0.1	3	0.5	0.5	0.1	0.1

Table 2 Comparison of final attitude with various numbers of rotations *N*

N	$V(\epsilon_N)$		
20	0.0564	0.2982	−0.0405
40	0.0393	0.3196	−0.0206
80	0.0276	0.3307	−0.0104

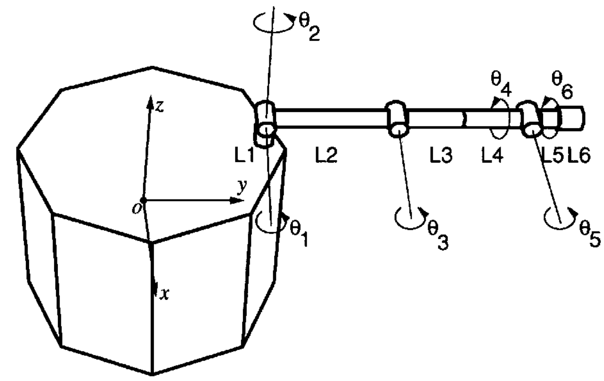

Fig. 2 Space robot model.

The difficulty of attitude control by an arm depends on the mass ratio of the arm and the satellite, the arm degrees of freedom, the initial arm posture, and the desired direction of the attitude change. All of these are reflected in the magnitude of vectors $\boldsymbol{a}_0$ and $\boldsymbol{b}_0$, which is a suitable measure of the difficulty. Because the characteristic $\|\boldsymbol{a}_0\| = \|\boldsymbol{b}_0\|$ holds for the method in Ref. 4, $\|\boldsymbol{a}_0\|$ is the magnitude of the arm motion. If this value is too large, we have to increase the number of rotations N. When $V(\gamma)$ is very small, $V(\gamma)$ is in inverse proportion to N. Then the values of $\|\boldsymbol{a}_0\|$ and $\|\boldsymbol{b}_0\|$ also decrease inversely as the square root of N from Eq. (20). In this example, $\|\boldsymbol{a}_0\| = \|\boldsymbol{b}_0\|$ is 0.4697 when N equals 20.

The values of $\boldsymbol{a}_0$ and $\boldsymbol{b}_0$ are set from the nominal values of parameters shown in Table 1, whereas in the simulations the mass of the arm tip (L6 mass) is set for three cases: 10, 20 (nominal value), and 40 kg. The control gains are set for two cases: $k_P = 0.5$, $k_I = 0.1$, and $k_F = 0.5$ (with feedback) and $k_P = 0$, $k_I = 0$, and $k_F = 0$ (without feedback).

The final satellite attitudes at the end of the Nth rotation with the values of $V(\epsilon_N)$ in the case of $T = 5$ s and $\upsilon = 1$ s are shown in Fig. 3. Thus, the values of $V(\hat{\epsilon}_N)$ [the desired values of $V(\epsilon_N)$] become

$$V(\hat{\epsilon}_N) = [0 \quad \sin(40/2) \quad 0]^T = [0 \quad 0.3420 \quad 0]^T \tag{46}$$

The values of $V(\epsilon_N)$ in Fig. 3 indicate the x, y, and z components from the top. As shown in Fig. 3, without feedback control, an attitude error occurs even when the tip mass is the nominal value. This is because Eq. (14) is an approximation for an infinitely small arm motion. For reference, Table 2 shows the final attitude without feedback control when N equals 20, 40, and 80. The attitude error decreases as N increases because the arm motion becomes smaller. Figure 3 also shows that the error becomes larger as the tip mass deviates farther from the nominal one. On the other hand, the final satellite attitude converges to the desired value for all cases with feedback control.

Figures 4 and 5 show the motion of the satellite attitude and that of the joint angles when the tip mass is the nominal value

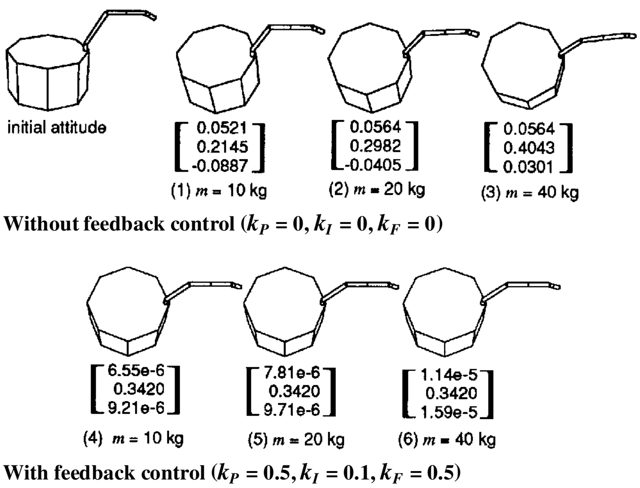

Fig. 3 Final attitude of space robot with/without feedback control; $N = 20, m$: tip mass.

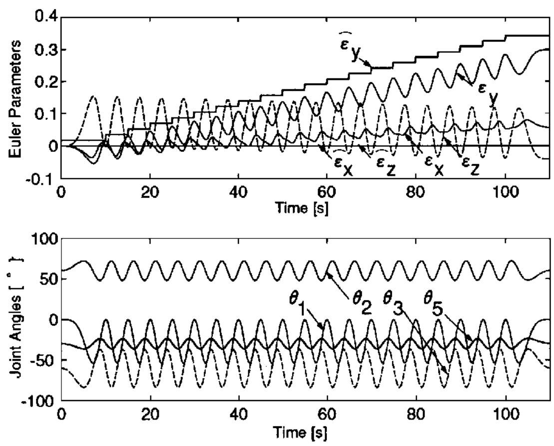

Fig. 4 Euler parameters and joint angles without feedback control.

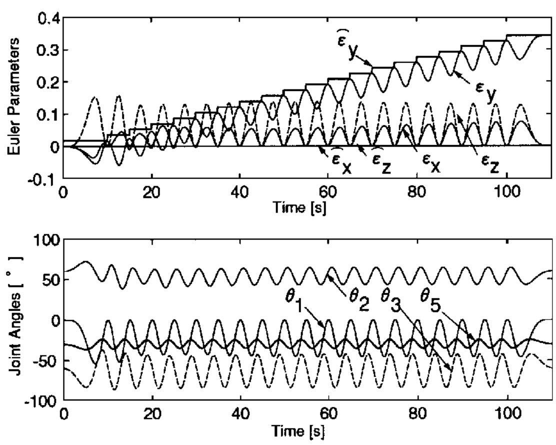

Fig. 5 Euler parameters and joint angles with feedback control.

without feedback control and with feedback control, respectively. The motion of the satellite attitude is shown by the components of $V(\epsilon)$ (ε_x, ε_y, ε_z), whereas those of $V(\hat{\epsilon}_k)$ ($\hat{\varepsilon}_x$, $\hat{\varepsilon}_y$, $\hat{\varepsilon}_z$) are also shown for reference. With feedback control the components of $V(\epsilon)$ move almost along the desired values, whereas without feedback control the error is noticeable. The joint angles are shown for θ_1, θ_2, θ_3, and θ_5, which change a lot. In case of feedback control, the movements of the joint angles are not always the same in each rotation because the values of $\boldsymbol{a}$ and $\boldsymbol{b}$ change, particularly at the beginning. From these results, it is obvious that feedback control is effective in decreasing the attitude error.

Conclusion

Focusing on the attitude control problem of a space robot using its arm rotations, we proposed a method of decreasing the attitude error by using feedback control. The method used derives an arm path to approximate the desired attitude change by plural rotations of the arm and then modifies it by feeding back the attitude error at the end of each rotation. We can use this method to obtain the desired attitude change even when modeling errors exist in the space robot model. When the angular momentum of the space robot is conserved at a nonzero value, the attitude change depends on the arm motion speed.[7] Thus, the problem of the remaining angular momentum is beyond the scope of this paper's method and is left for future work.

Appendix: Euler Parameters

Euler parameters[9] are a kind of quaternion composed of a scalar part and a three-dimensional vector part and are used to express attitude. We assume here that the directions of a coordinate system A coincide with those of a coordinate system B after rotating the coordinate system A by an angle ψ around a unit vector $\boldsymbol{e}$. Thus, the Euler parameters ϵ_{BA} that express the attitude of the coordinate system B to the coordinate system A are expressed as

$$\epsilon_{BA} = \begin{bmatrix} \boldsymbol{e}\sin(\psi/2) \\ \cos(\psi/2) \end{bmatrix} \tag{A1}$$

where vector $\boldsymbol{e}$ is expressed in the coordinate system A or B (the results are the same in both coordinate systems). Taking one more coordinate system, C, we can express ϵ_{CA} with ϵ_{BA} and ϵ_{CB} as

$$\epsilon_{CA} = \begin{bmatrix} S(\epsilon_{BA})\boldsymbol{I}_3 + V^{\times}(\epsilon_{BA}) & V(\epsilon_{BA}) \\ -V^T(\epsilon_{BA}) & S(\epsilon_{BA}) \end{bmatrix} \epsilon_{CB} \tag{A2}$$

where $V^{\times}$ and V^T designate a 3×3 matrix expressing a vector product operation of V and a transpose of V, respectively. Regarding the operation of the right-hand side of Eq. (A2) as a product between two sets of Euler parameters, we abbreviate it as follows:

$$\epsilon_{CA} = \epsilon_{BA}\epsilon_{CB} \tag{A3}$$

Euler parameters whose vector part has the opposite sign to that of the original Euler parameters are expressed as the original Euler parameters with a superscript +. The superscript + corresponds to the reciprocals of Euler parameters, that is,

$$\epsilon^{+}_{BA}\epsilon_{BA} = \epsilon_{BA}\epsilon^{+}_{BA} = [0 \quad 0 \quad 0 \quad 1]^T \tag{A4}$$

where the right-hand side of Eq. (A4) is a unit element of the Euler parameters.

A coordinates transformation matrix $\boldsymbol{U}_{BA}$ that transforms the expression in the coordinate system A into that in the coordinate system B is expressed by Euler parameters ϵ_{BA} as

$$\boldsymbol{U}_{BA} = \left\{S(\epsilon_{BA})^2 - V^T(\epsilon_{BA})V(\epsilon_{BA})\right\}\boldsymbol{I}_3 + 2V(\epsilon_{BA})V^T(\epsilon_{BA}) - 2S(\epsilon_{BA})V^{\times}(\epsilon_{BA}) \tag{A5}$$

Let the angular velocity of the coordinate system B to the coordinate system A be $\boldsymbol{\omega}_{BA}$, and let the expressions of $\boldsymbol{\omega}_{BA}$ in the coordinate systems A be ${}^{A}\boldsymbol{\omega}_{BA}$. Thus, the time derivative of Euler parameters ϵ_{BA} has a relation with ${}^{A}\boldsymbol{\omega}_{BA}$ of

$$\dot{\epsilon}_{BA} = \frac{1}{2}\begin{bmatrix} S(\epsilon_{BA})\boldsymbol{I}_3 - V^{\times}(\epsilon_{BA}) \\ -V^T(\epsilon_{BA}) \end{bmatrix} {}^{A}\boldsymbol{\omega}_{BA} \tag{A6}$$

The 4×4 matrix $\Lambda(\boldsymbol{v})$ is defined with an arbitrary 3×1 vector $\boldsymbol{v}$ as

$$\Lambda(\boldsymbol{v}) = \begin{bmatrix} \boldsymbol{v}^{\times} & \boldsymbol{v} \\ -\boldsymbol{v}^T & 0 \end{bmatrix} \tag{A7}$$

where $\boldsymbol{v}^{\times}$ designates a 3×3 matrix expressing a vector product operation of $\boldsymbol{v}$. Then Eq. (A6) can be changed into the following form:

$$\dot{\epsilon}_{BA} = \tfrac{1}{2}\Lambda\left({}^{A}\boldsymbol{\omega}_{BA}\right)\epsilon_{BA} \tag{A8}$$

For a very small 3×1 vector $\boldsymbol{w}(\|\boldsymbol{w}\| \ll 1)$, it is expressed as Euler parameters $E(\boldsymbol{w})$, that is,

$$E(\boldsymbol{w}) = \begin{bmatrix} \boldsymbol{w} \\ \sqrt{1 - \boldsymbol{w}^T \boldsymbol{w}} \end{bmatrix}, \qquad E^+(\boldsymbol{w}) = \begin{bmatrix} -\boldsymbol{w} \\ \sqrt{1 - \boldsymbol{w}^T \boldsymbol{w}} \end{bmatrix} \tag{A9}$$

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